Summer *AP* Assignment Coversheet

Falls Church High School

Course: AP Calculus BC

Teacher Name/s: Marla Schnall

Assignment Title: AP Calculus BC Summer Packet

Assignment Summary/Purpose:

The material in this packet should be a review of past material that will be necessary in BC Calculus. Completion of this packet will set each student up for continued success in the AP Calculus program.

Due date (must be completed before school starts)

Assigned during:	Summer break
Due Date:	8/30/2019

Estimated time needed to complete the assignment:
Approximately 10 hrs.

Description of how the assignment will be assessed:

The material from this packet will be used throughout the course and will be assessed as part of unit tests and quizzes.

Grade impact to overall course grade:

Content will be integrated into the coursework.

Tools/resources needed to complete the assignment:

TI-84 graphing calculator

Contacts:

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Falls Church High School
Mathematics Department

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PUBLIC SCHOOLS

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Dear 2019-2020 AP Calculus Student,

Congratulations on your decision to explore the world of Calculus this year. Attached to this letter you will find a packet of materials for you to work on over the summer. This packet is designed to ensure that you can recall and apply many of the wonderful algebra, geometry, and trigonometry skills that you have learned in the past few years. While calculus is an entirely new subject to you, it is strongly based on algebra and trigonometry. Your success in calculus depends strongly on your ability to recall and apply the skills that you have learned in prior math courses.

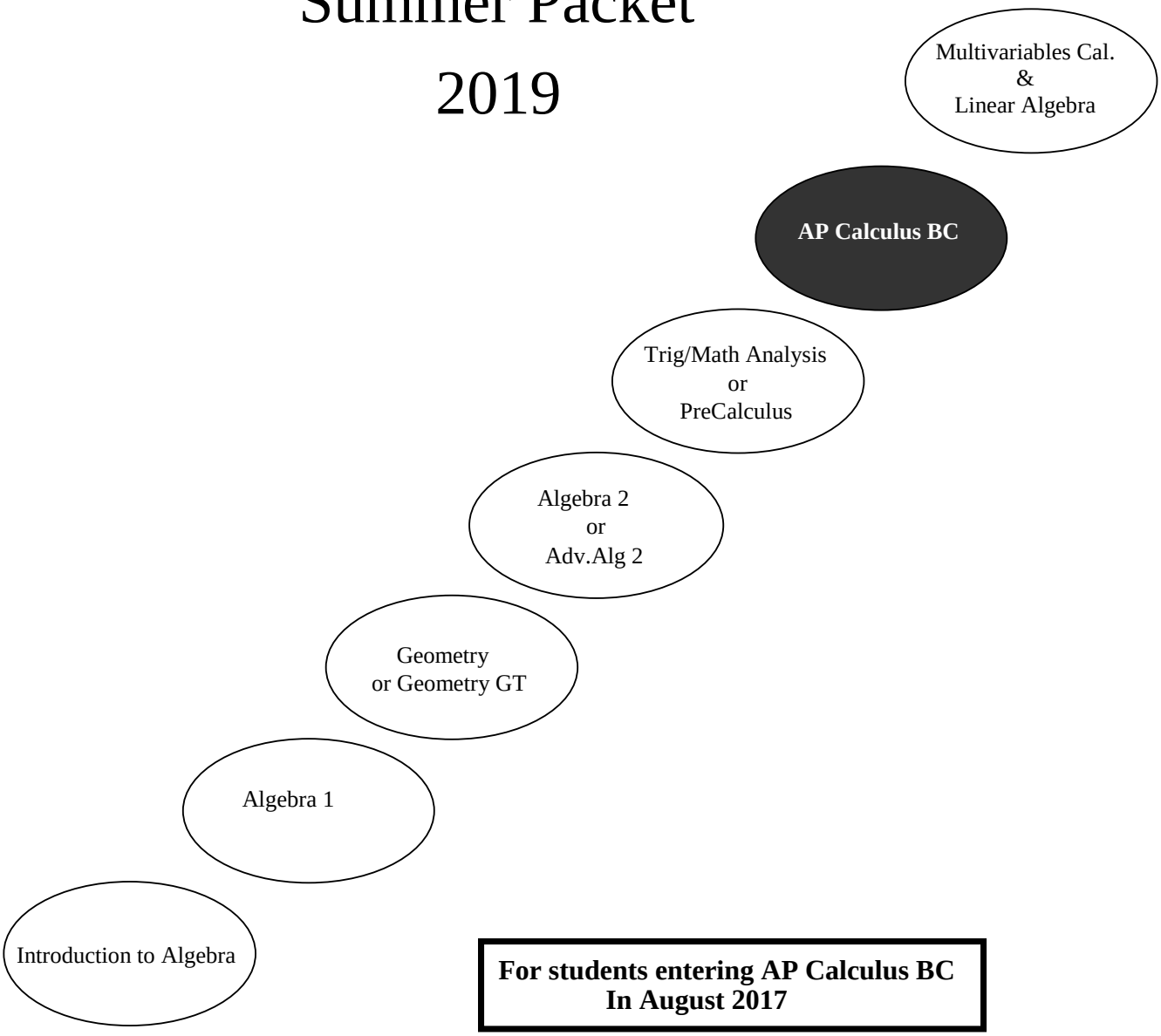
You will have the opportunity to ask questions about anything in it during the first week of class. I do not expect you to spend more than 10 hours to complete this assignment. Your readiness for this course will be determined by how much information you retained from your previous math courses and your ability of applying them in real-life situation to solve problems. Therefore, the completion of this assignment will prepare you for the first weeks of Calculus. I will use this assignment to plan review and remediation as necessary.

You will need to get your own graphing calculator for next year. You should get a Ti-84 plus with the most up-to-date operating system. The Ti-89 is also allowed for the course, but you will only receive instruction on how to use the Ti-84.

Calculus is challenging and sometimes fun and definitely different from anything you have studied before. As an AP student, I expect you to work hard and to take the initiative to find out about things that you don't understand. I hope you enjoy your summer and look forward to seeing you in August 2019!

Mrs. Schnall

Falls Church High School
Mathematics Department
Summer Packet
2019



For students entering AP Calculus BC
In August 2017

FUNCTIONS

A **function** is a rule that assigns to each member in its **domain** a unique member in the **range.**
Domain: The possible x-values of a function
Range: The possible y-values of a function
FUNCTION NOTATION: f(x)

To be written as f(x) means the represented expression satisfies the definition of a function, i.e. for each x-value, there is an unique corresponding y-value

<u>EVALUATION OF FUNCTIONS</u>:

f(a) means replace each 'x' in f(x) with 'a' to find the function's y-value

EXAMPLE:
f(x) = 2x – 5
f(3) = 2(3) – 5
f(3) = 6 – 5
f(3) = 1

<u>ADDITION, SUBTRACTION AND MULTIPLICATION OF FUNCTIONS</u>:

Functions can be added, subtracted and multiplied just as any other expressions:
EXAMPLE: $f(x) = 3x + 2$ $\quad g(x) = x^2 + x - 1$

$$f(x) + g(x) = x^2 + 4x + 1$$
$$f(x) - g(x) = -x^2 + 2x + 3$$
$$f(x) \bullet g(x) = 3x^3 + 5x^2 - x - 2$$

<u>COMPOSITION OF FUNCTIONS</u>:
Composition of functions means instead of replacing 'x' in a function with a number, an entire function is inserted instead.
EXAMPLE: $f(x) = x - 3$ $\quad g(x) = x^2 + 2$
The composition of f(x) and g(x) is written: $f(g(x))$
To evaluate, simply plug in g(x) wherever an 'x' appears in f(x).

$$f(x) = x - 3$$
$$f(g(x)) = (x^2 + 2) - 3$$
$$= x^2 - 1$$

$$g(x) = x^2 + 2$$
$$g(f(x)) = (x - 3)^2 + 2$$
$$= x^2 - 6x + 9 + 2$$
$$= x^2 - 6x + 11$$

<u>Function Problems:</u>

1. Find the *x-intercepts* and the *y-intercepts* of $f(x) = x^2 - 12x + 32$

2. Find the domain and range of the function. $h(x) = \dfrac{13}{x+2}$

3. What is the domain of the function $f(x)=\ln(x^2-4)$?

4. Identify the domain of the function $f(x)=\sqrt{5-x^2}$, and find $f^{-1}(x)$..

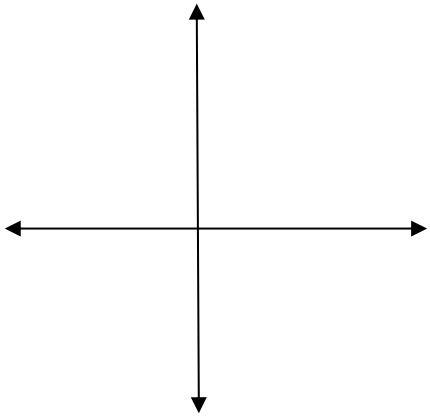
5. Given $f(x)=-|x-3|-2$

   a. Domain: _____ Range: _____

   b. f(3) = ____ f(x+5) = _____

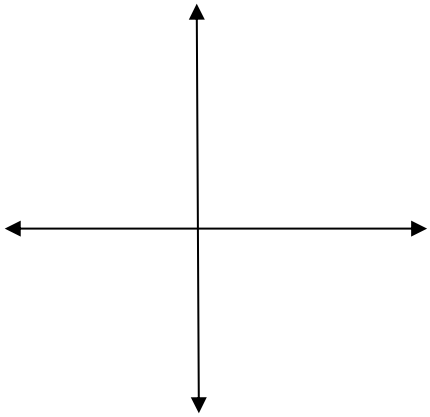
   c. f(x) = -3 then x = ____

   d. Sketch f(x)



6. a. Graph the piece-wise function

$$g(x)=\begin{cases}\dfrac{x}{2}, x\geq 4\\ \sqrt{x}, 0<x<4\\ x^2, x\leq 0\end{cases}$$

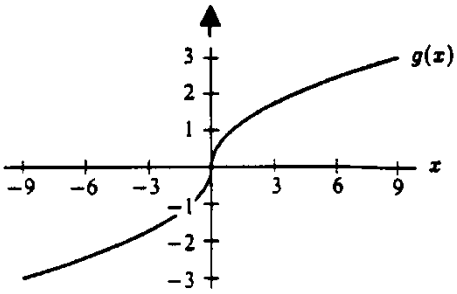


   b. Identify the domain and the range of g(x)

   c. g(-3) = ____ g(1) = ____ g(0) = ____

   d. Is g(x) a continuous function? ____

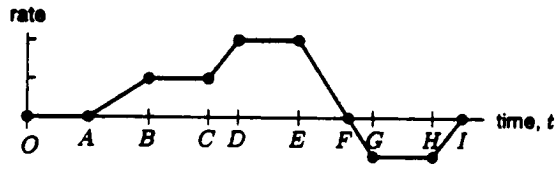
6. Given the graph of g(x) on the right



   a. Estimate $\dfrac{g(6)-g(0)}{6-0}=$

   b. The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment.

7. The rate at which water is entering a tank (t > 0) is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:



a. The volume of water is constant.

b. The volume of water is decreasing.

c. The volume of water is increasing.

d. The volume of water is increasing fastest.

8. $Q(x) = \dfrac{3x}{x+1}$

a. What is the domain of Q(x)?

b. Where is this function discontinuous? ___

c. State the equation of the vertical asymptote x = ___

d. State the equation of the horizontal asymptote y =___

e. Sketch the graph.

f. Write the equation of the inverse of Q(x) (Switch the x & y and then rewrite as y =)

9. Use these functions: $f(x) = x + 1$ $g(x) = x^2 + 2x - 3$ $h(x) = 2x - 5$

a) $f(3) =$

b) $g(5) =$

c) $h(\frac{1}{2}) =$

d) $g(-2) =$

e) $f(x) + h(x) =$

f) $g(x) - h(x) =$

g) $f(h(1)) =$

h) $h(g(-3)) =$

i) $h(f(x)) =$

j) $g(f(x)) =$

TRIGONOMETRY

What you need to know:

- Trig functions and inverse trig functions for all special angles (unit circle)
- Fundamental trig identities (reciprocal, quotient, Pythagorean)
- Graphs of sin, cos, tan, sec, csc
- Domain and range of sin, cos, tan, sec, csc
- Solve trig equations

Trigonometric Identities

$$\sin^2(x)+\cos^2(x)=1$$

$$\tan^2(x)+1=\sec^2(x)$$

$$\cot^2(x)+1=\csc^2(x)$$

$$\tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$\cot(x)=\frac{\cos(x)}{\sin(x)}$$

$$\sec(x)=\frac{1}{\cos(x)}$$

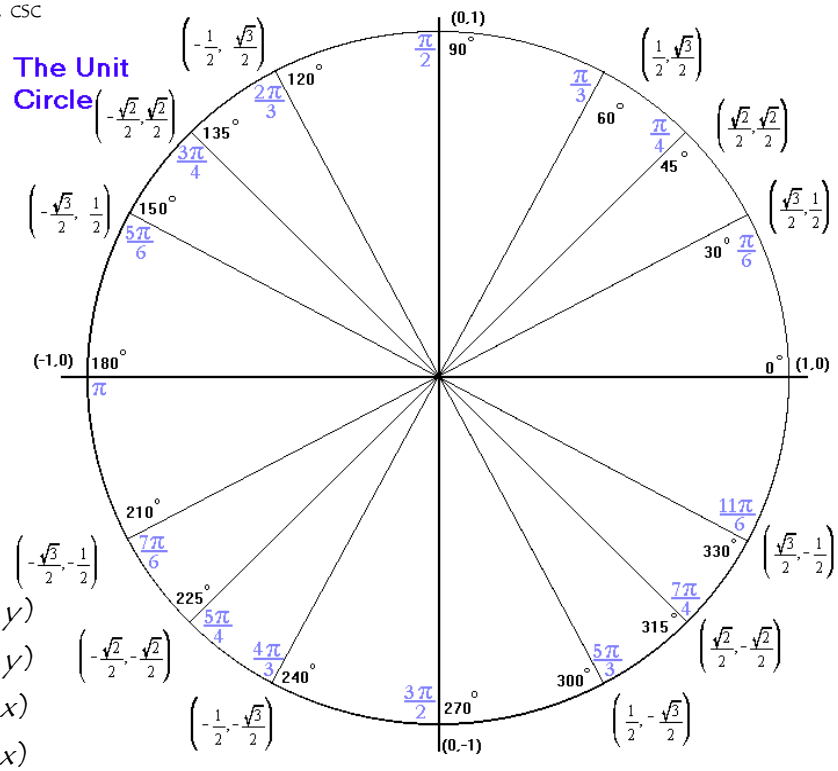
$$\csc(x)=\frac{1}{\sin(x)}$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$

$$\sin(x+y)=\sin(x)\cos(y)+\sin(y)\cos(x)$$

$$\sin(x-y)=\sin(x)\cos(y)-\sin(y)\cos(x)$$



1. Evaluate without use of a calculator.

(a) $\tan(\frac{\pi}{6})=$ (b) $\sin(\pi)=$ (c) $\cos(\frac{-\pi}{3})=$ (d) $\csc(\frac{\pi}{2})=$

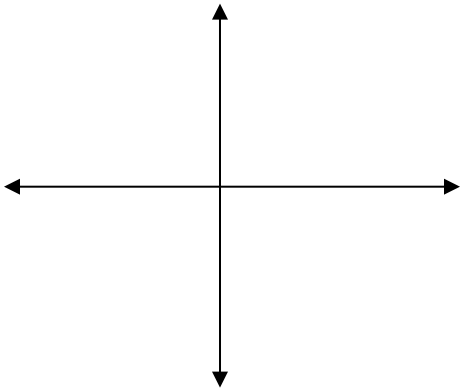
2. Find the exact values without use of a calculator.

(a) $\sin^{-1}(\frac{\sqrt{3}}{2})=$ (b) $\arctan(-\sqrt{3})=$ (c) $\cos^{-1}(\frac{1}{2})=$

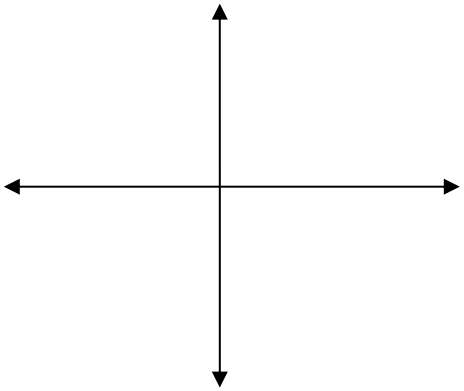
(d) $\sec^{-1}(-2)=$ (e) $\arcsin(-\frac{\sqrt{2}}{2})=$ (f) $\sin(\cos^{-1}0.6)=$

3. Sketch the graphs of the following without the use of a calculator. Show at least two periods:

(a) $y = \cos \pi x - 1$



(b) $y = 2\tan(x - \frac{\pi}{4})$



4. Solve the following equations for x.

a. $\cos^{-1}(7x - \pi) = \frac{1}{2}$

b. $\sec(x) = \tan(x) + \cot(x)$

c. $\sqrt{3}\sin x + \cos x = 1$

d. $\sin^2(x) = 2\cos(x) + 2$

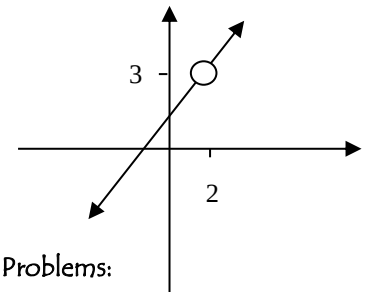
e. $\tan(2x) + 2\sin x = 0$

f. $2\sin^2 x - \sin x - 1 = 0$

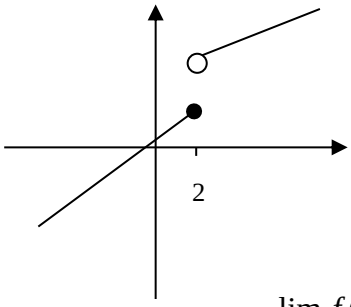
5. Write the following expression in algebraic form. $\sin(\cos^{-1}(2x))$

Limits

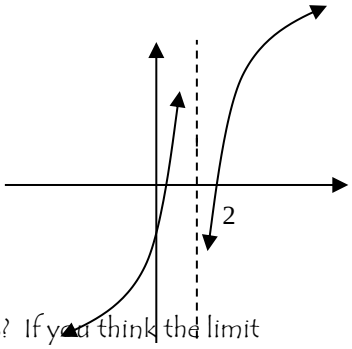
The limit of a function is the <u>y-value</u> that you are getting close to as x gets close to some number in the domain. In the "limit process" you never get to the limit, except for the limit of a constant function. We write $\lim_{x \to a} f(x)$, which is read "the limit of f(x) as x approaches a domain value of a." The limit must be the same as x approaches "a" from both the left and the right. To find the limit, substitute in values very close to "a" on both left and right and see if the y-value is approaching a single value. The limit does exist at a hole in a graph, but does not exist at a vertical asymptote or a jump in the graph.



Limit exist at 2



No limit at 2

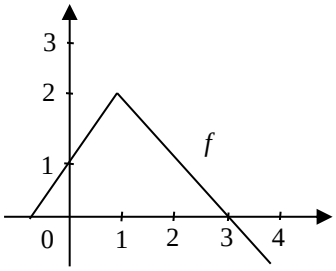


No limit at 2

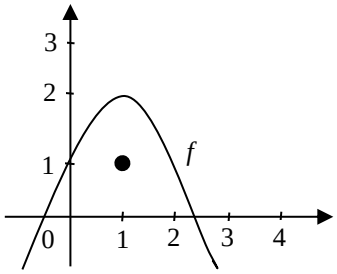
Problems:

The s s are pictured below. $\lim_{x \to 1} f(x)$ exists? If you think the limit does exits, state its value.

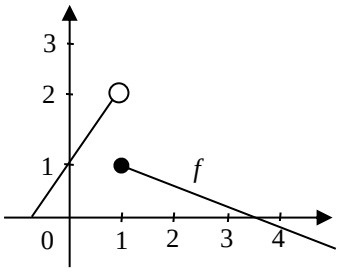
1.



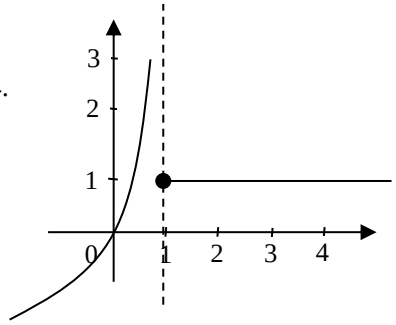
2.



3.



4.



State the value of each of the following:

5. $\lim_{x \to 3} x$

6. $\lim_{x \to -2} x$

7. $\lim_{x \to 1} 2x$

8. $\lim_{x \to -1} (x+1)$

9. $\lim_{x \to -2} x^2$

10. $\lim_{x \to -2} |x|$

11. Find the limit $\lim_{x \to \frac{2\pi}{3}} \sin x$

12. Let $f(x) = -x^2 + 3$ and $g(x) = 2x$. Find the limits:

(a) $\lim_{x \to -3} f(x)$ (b) $\lim_{x \to -4} g(x)$ (c) $\lim_{x \to -1} g(f(x))$

13. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point. $\lim_{x \to -6} \frac{x^3 + 216}{x + 6}$

14. Use a graphing calculator to complete the table and use the result to estimate the limit.

$$\lim_{x \to -7} \frac{\sqrt{2x + 22} - \sqrt{8}}{x + 7}$$

X	–7.1	–7.01	–7.001	–6.999	–6.99	–6.9
f(*x*)						

THE EQUATIONS OF A LINE

Write the equation of the line described in the form y = mx + b. Where $m = \frac{y_2 - y_1}{x_2 - x_1}$ is the slope of the line and b, the y-intercept.

The point slope form which is given by $y - y_1 = m(x - x_1)$ is the most used in Calculus. A point *(x_1, y_1)* and the slope *m* are known or given.

Example: The line through (3. 6) and (-9, -2)

Solution: $m = \frac{-2-6}{-9-3} = \frac{2}{3}$

$$(y - y_1) = m(x - x_1)$$

$$y - 6 = \frac{2}{3}(x - 3)$$

$$y = \frac{2}{3}x + 4$$

1. The line through (1, 4) and (3, 6)

2 .The line through (5, -2) and (-5, 4)

3. The line through (2, 1) with slope 4

4. The line with slope $\frac{7}{2}$ and passing through (-2, -5)

5. The line with slope 8 and y-intercept 9

6. The line with slope $-\frac{3}{8}$ passing through (0, 5)

7. The line through (-2, -8) and parallel to the line y = 5x - 3

8. The line perpendicular to y = $\frac{4}{5}$x - 9 and passing through (8, -13)

EXPONENTS

Properties of exponents

$x^m * x^n = x^{m+n}$ $\qquad \frac{x^m}{x^n} = x^{m-n}$ $\qquad \left(x^m\right)^n = x^{m*n}$

$\sqrt[n]{x^m} = x^{m/n}$ $\qquad x^{-m} = \frac{1}{x^m}$ $\qquad x^0 = 1$

$(ab)^x = a^x b^x$ $\qquad (\frac{a}{b})^x = \frac{a^x}{b^x}$

$\log_a(xy) = \log_a x + \log_a y$ $\qquad \log_a(\frac{x}{y}) = \log_a x - \log_a y$

$\log_a x^y = y \log_a x$

$\log_a x = \frac{\log_b x}{\log_b a}$

Completely simplify each expression

1. $2\left(x^4 y^3\right)^0$

2. $\frac{8^{-2} * 4^{-2}}{16^{-2}}$

3. $\frac{3c^2 d^3}{\left(3cd^{-2}\right)^2}$

4. $\frac{3^7 * 9^5}{\sqrt{27^{12}}}$

5. $(32)^{-2/5}$

6. $\sqrt{x} * \sqrt[3]{x} * \sqrt[4]{x}$

7. $\frac{x^2 - x + 7}{x}$

8. $\frac{x^3 - x + 1}{\sqrt{x}}$

9. $x^{3/2}\left(x^2 - 2x\right)$

SOLVING EQUATIONS

SOLVE FOR X.

1. $4(x+3)-3=2(4-3x)-4$

2. $15+x-2x^2=0$

3. $5x^4-12x^3=0$

4. $\dfrac{1}{x-2}=3$

5. $4t^3-12t^2+8t-24=0$

6. $\dfrac{4}{x-3}-\dfrac{4}{x}=1$

7. $\sqrt{x-2}-8=0$

8. $(x+2)^{3/4}=27$

9. $|x^2-6|=x$

10. $\left|x-\dfrac{3}{2}\right|\le 1$

11. $3e^{-5x}=132$

12. $e^{2x}-7e^x+10=0$

13. $\ln x-\ln 3=2$

14. $2\ln 4x=15$

15. $2\cos x+1=0$

16. $3\sec^2 x-4=0$

17. $\sin^2 x=3\cos^2 x$

18. $\log_2 x+\log_2 5=6$

19. $3s^2+6s+2=0$

20. $3^{5x+3}=81$

21. $\ln(x-2)^3=2$

22. Write the expression $2\ln(3)-\frac{1}{5}\ln\left(x^2+4\right)$ as the logarithm of a single quantity

<u>Sequences and Series</u>

$a_n = 4n + 3$

1. What is the 4th term of the sequnce?

2. What is the (n+1)st term of the series?

3. What is the difference between the nth term and the (n+3)rd term?

$$a_n = \left(\frac{3}{5}\right)^n$$

4. What is the 6th term of the sequence?

5. What is the sum of the first 3 terms

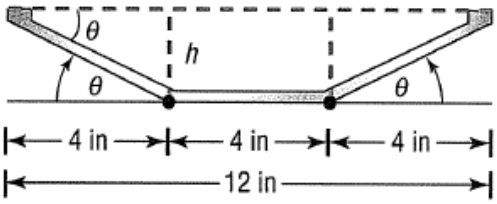
6. Evaluate the sum $\sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n$

7. $3+\frac{3}{7}+\frac{3}{49}+\ldots\ldots\ldots$

Constructing a Rain Gutter

A rain gutter is to be constructed of aluminum sheets 12 in wide. After marking off length of 4in from each edge, this length is bent up at an angle θ.

a- Show that the area A of the opening or cross-section (as shown on the illustration) of the gutter as a function of θ is given by $A(\theta) = 16\sin\theta(\cos\theta + 1), 0 < \theta < \frac{\pi}{2}$



b- In Calculus, you will be asked to find the angle θ that maximizes A by solving the equation $\cos(2\theta) + \cos\theta = 0$. Solve this equation for θ by using the Double-Angle Identity

c- Solve the equation for θ by writing the sum of the two (2) cosines as a product.

d- What is the maximum area A of the opening?